

Indian Statistical Institute  
Semestral Examination - M.Math II  
Algebraic and Differential Topology

Time : 3 hours

12.12.2008

Max. Marks = 60.

Answer all questions. State clearly any result that you use.

1. (a) Define the term *fibration*. Show by an example that the fibers over two distinct points of a fibration (with connected base and total space) need not be homeomorphic.  
(b) Show that  $\pi_2(S^2 \vee S^1)$  is not finitely generated. [8+7]
2. (a) Let  $M$  be a smooth manifold of dimension at least 1 and let  $p \in M$ . Show that there exists a smooth function  $f : M \rightarrow \mathbb{R}$  that is a submersion at  $p$ .  
(b) Show that  $SL(n, \mathbb{R})$ , the space of  $n \times n$  real matrices with determinant 1, is a smooth manifold. Describe the tangent space to  $SL(n, \mathbb{R})$  at the identity matrix. [8+7]
3. (a) Let  $M$  and  $N$  be connected smooth manifolds of dimension  $n$  with  $M$  compact. Let  $f : M \rightarrow N$  be a submersion. Show that  $f$  is a covering map.  
(b) Show that there does not exist an immersion  $g : S^1 \times S^1 \rightarrow \mathbb{R}P^2$ .  
(c) Does there exist a smooth one-one map  $h : \mathbb{R}^2 \rightarrow \mathbb{R}$ ? [8+4+3]
4. (a) Let  $(r, \theta)$  denote the polar coordinates on  $\mathbb{R}^2 - 0$ . Describe the 1-form  $d\theta$  in cartesian coordinates. Show that  $d\theta$  is not exact.  
(b) Compute  $H_{dR}^1(S^1)$ . [8+7]